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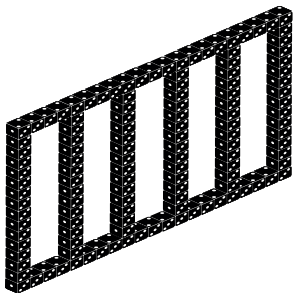
Team Round, 3 problems / 1 hour / 210 points

October 21, 2023

Problem 1 (Party)**.** Mo has invited 2023 guests for his retirement party. His way of the sharing the cake is quite eccentric: the first guest gets $\frac{1}{2023}$rd of the cake, the second guest gets $\frac{2}{2023}$rd of what is left, the third guest gets $\frac{3}{2023}$rd of what is left, ..., and the last guest gets $\frac{2023}{2023}$rd -that is everything- of what is left.
Which guest receives the largest piece?

Problem 2 (Alea iacta est)**.** Let Δ represent the difference between the largest possible sum and the smallest possible sum of all visible faces on a dice configuration. Imagine a construction like the one below but where the number of 'holes' is not 5 but some larger number g. If $\Delta = 2032$ for that construction, what is the number of holes (g)?



Problem 3 (This problem stinks)**.** The septic number system consists of the positive integers of the form $7n + 1$: that is, 1, 8, 15, 22, etc. A septic prime is a septic number larger than 1 that cannot be written as a product of two smaller septic numbers. Every septic number larger than 1 can be written as a product of septic primes, but this factorization is not always unique. For example, $36 \times 169 = 78 \times 78$, and all of 36, 169, and 78 are septic primes. In this instance our two factorizations have length 2, where the length is the number of septic primes involved in the factorization (with repeated primes counted multiply).

For each septic integer n, let

$$E(n) = \frac{\text{largest length of a factorization of } n \text{ into septic primes}}{\text{smallest length of such a factorization}}.$$

Find the largest possible value of $E(n)$.