Sponsored by: UGA Math Department and UGA Math Club

TEAM ROUND / 1 HOUR / 210 POINTS
October 22, 2022

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1 (Going in circles)**.** What is the smallest radius r so that 3 disks of raidus r can completely cover a disk of raidus 1?

Problem 2 (Making a difference)**.** Suppose a, b, c, and d are distinct positive integers satisfying

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} < 1.$$

Define the difference D as

$$D = 1 - \frac{1}{a} - \frac{1}{b} - \frac{1}{c} - \frac{1}{d},$$

and write $D = r/s$ in lowest terms. If a, b, c, and d are chosen so that D is as small as possible, what is $r + s$?

Problem 3 (Descent into madness)**.** How many equilateral triangles can be formed using the integer points which lie in the cube $[0,4] \times [0,4] \times [0,4]$? Note: The integer points on the surface of the cube are also included for a total of 125 integer points.