Sponsored by: UGA Math Department and UGA Math Club

TEAM ROUND / 1 HOUR / 210 POINTS
October 26, 2019

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1 (Cold-blooded mathematics). Recall that an object is an n-*reptile* if it can be decomposed into n congruent pieces each similar to the original figure.

If a right triangle with shortest leg 1 is a 5-reptile, what is the length of the hypotenuse?

Problem 2 (Colors and numbers). If the positive integers from 1 to 30 are all colored the same color, then there are guaranteed to be numbers x, y, z that are all the same color and satisfy $x+y = z$ — a "monochromatic solution to $x + y = z$". At the other extreme, if the positive integers from 1 to 30 are colored 30 different colors, then there are no monochromatic solutions to $x + y = z$. What is the smallest integer n for which it is possible to color 1 to 30 with n colors and have no monochromatic solution to $x + y = z$?

Note: We do *not* require that x, y, z be distinct. That is, a solution to $x + y = z$, where $x = y$, and where x and z share the same color, counts as a monochromatic solution.

Problem 3 (Unscrambling an egg). The average of a set of integers is computed by taking the sum of the elements divided by the total number of elements. For example, the average of the set $\{1,5\}$ is $\frac{1+5}{2} = 3$ and the average of the set $\{1,5,6\}$ is $\frac{1+5+6}{3} = 4$.

Let A be a set with 7 elements (so A has 127 nonempty subsets). The averages of all of the 127 subsets of A are listed below, in increasing order. What are the 7 elements of A?

Write the numbers you find in increasing order. *You must have all the numbers correct to receive credit for this problem.*

#	val	#	val	#	val	#	val	#	val
1	759	27	4014	53	5043	79	5659	105	6723
2	969	28	4119	54	5064	80	5694	106	6744
3	1179	29	4119	55	5099	81	5799	107	6779
4	1319	30	4224	56	5127	82	5799	108	6807
5	1389	31	4259	57	5169	83	5799	109	6919
6	1599	32	4259	58	5169	84	5883	110	6975
7	2019	33	4287	59	5169	85	5904	111	7059
8	2334	34	4329	60	5211	86	5939	112	7164
9	2439	35	4329	61	5211	87	6009	113	7199
10	2719	36	4371	62	5239	88	6009	114	7374
11	2859	37	4399	63	5239	89	6009	115	7479
12	2964	38	4434	64	5259	90	6009	116	7619
13	3069	39	4469	65	5274	91	6051	117	7689
14	3279	40	4539	66	5295	92	6079	118	7759
15	3279	41	4539	67	5379	93	6114	119	7899
16	3384	42	4539	68	5379	94	6135	120	8214
17	3447	43	4539	69	5379	95	6219	121	8319
18	3489	44	4644	70	5379	96	6219	122	8739
19	3559	45	4679	71	5379	97	6219	123	8949
20	3699	46	4707	72	5379	98	6324	124	9159
21	3699	47	4819	73	5379	99	6359	125	9579
22	3783	48	4819	74	5519	100	6429	126	9789
23	3804	49	4854	75	5547	101	6499	127	9999
24	3839	50	4959	76	5589	102	6534		
25	3867	51	4959	77	5589	103	6639		
26	3979	52	4959	78	5631	104	6639		

RETURN THIS SHEET

Team ID:

Team name:

Answer 1:

Answer 2:

Answer 3: