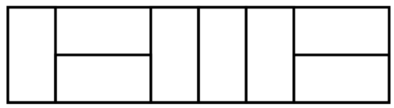
Problem 1. A $2 \times N$ room is tiled with small rectangles of size 1×2. An example of tiling is given in the picture below. If $N = 8$, in how many ways can you tile the room?



ANSWER:

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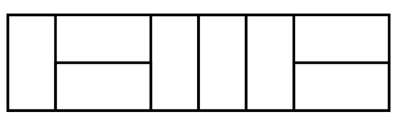


ANSWER:

Problem No.1

NAME

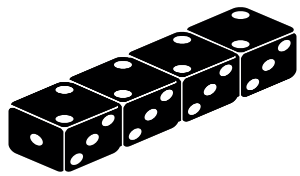
ID

Problem No.1

NAME

ID

Problem 2. Four identical dice are stacked like on the picture below. The internal dice show four faces and the boundary ones five. What is the sum of the dots on the 18 visible faces? Recall that on a dice, opposite sides add up to 7.



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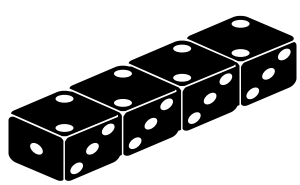

ANSWER:

PROBLEM NO.2

NAME

ID

PROBLEM NO.2

NAME

ID

Problem 3. Two points in the plane have integer coordinates and the distance between them is $\sqrt{65}$. If these points form the opposite corners of a rectangle whose sides are parallel to the coordinate axes, what is the maximal area of such a rectangle.

ANSWER:

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ANSWER:

Problem No.3

NAME

ID

Problem No.3

NAME

ID

Problem 4. What is the remainder of the following integer division?

$$\frac{1000^4 + 1001^4 + 1002^4 + \cdots + 2023^4}{16}$$

ANSWER:

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ANSWER:

Problem No.4

NAME

ID

Problem No.4

NAME

ID

Problem 5. In the expression below, a,b and c represent digits. If

$$0.2a \times 7.b = 2.c,$$

what is $a \times b \times c + a + b + c$?

ANSWER:

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$$0.2a \times 7.b = 2.c,$$

what is $a \times b \times c + a + b + c$?

ANSWER:

Problem No.5

NAME

ID

Problem No.5

NAME

ID

Problem 6. What is the last digit of M?

$$M = 1! + 2! + 3! + \cdots + 2023!$$

ANSWER:

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$$M = 1! + 2! + 3! + \cdots + 2023!$$

ANSWER:

Problem No.6

NAME

ID

Problem No.6

NAME

ID

Problem 7. Find the integer a that satisfies

$$a!(a+1)! = 10!$$

ANSWER:

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ANSWER:

PROBLEM NO.7

NAME

ID

PROBLEM NO.7

NAME

ID

Problem 8. How many triangles have sides of length π, $\sqrt{2}$ and a where the latter is an integer? (the order of the sides is irrelevant)

ANSWER:

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ANSWER:

Problem No.8

NAME

ID

Problem No.8

NAME

ID

Problem 9. Everytime a frog jumps, it makes a 1 foot leap in an arbitrary direction in the plane. What is the probability that after 2 leaps, the frog is within 1 foot from its starting position?

ANSWER:

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ANSWER:

Problem No.9

NAME

ID

Problem No.9

NAME

ID

Problem 10. If the integers a, b, c and d are such that

$$2^a 3^b 5^c 7^d - 77 = 2023,$$

what is $a + b + c + d$?

ANSWER:

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$$2^a 3^b 5^c 7^d - 77 = 2023,$$

what is $a + b + c + d$?

ANSWER:

Problem No.10

NAME

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Problem No.10

NAME

ID

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