TEAM ROUND / 45 MIN / 210 POINTS
October 17, 2009

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1. (Up or down?)   What is the smallest n such that in any sequence of n distinct numbers

$$a_1, a_2, a_3, \ldots, a_n$$

there is either an increasing subsequence of length 10 or a decreasing subsequence of length 10?

Example. In the sequence

$$1, 3, 2, 8, 4, 0, 5$$

a longest increasing sequence has length 4:

$$\boxed{1}, 3, \boxed{2}, 8, \boxed{4}, 0, \boxed{5}$$

and a longest decreasing sequence has length 3:

$$1, \boxed{3}, \boxed{2}, 8, 4, \boxed{0}, 5$$

Problem 2. (Doubling up) Find a positive integer which doubles when its last digit is moved in front. The number is to be written in standard decimal notation with no leading zeroes.

Example. The number 1234 becomes 4123 when its last digit is moved in front (so it doesn't work).

Problem 3. (Lucky horseshoe) A horseshoe has the shape of a semicircle of diameter 1. We throw it randomly on a square grid of 1 by 1 squares and count how many times it intersects the lines. After a very large number N of throws, the number of intersections will be close to cN for some number c. What is c?

Example. The following throw counts for three intersections:
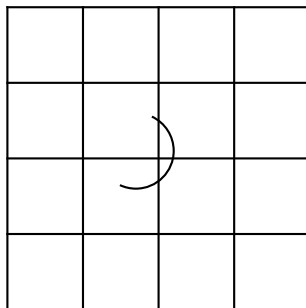
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RETURN THIS SHEET

Team ID:

Team name:

Answer 1:

Answer 2:

Answer 3: