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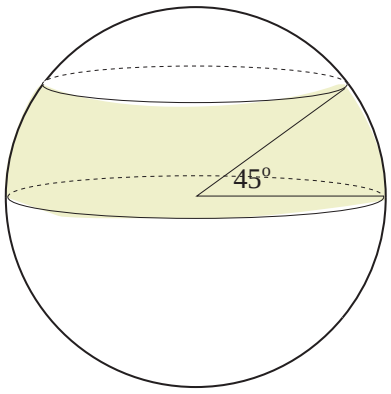
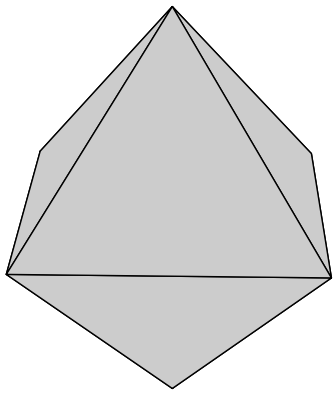
TEAM ROUND / 1 HOUR

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 50 points, for a total of 200 points.

Problem 1.(The globe) What portion of the earth's surface is located between the equator and the 45^0 latitude?



Problem 2.(Planets) 6 planets of radius 1 are centered at the vertices of a regular octahedron of side length 3. A point on the surface of one of the planets is called *invisible* if it cannot be seen from some other planet. What is the total area of the set of invisible points?



Problem 3.(Pascal's triangle) What are the first two digits after the decimal point in

$$(\sqrt{3}+\sqrt{2})^{2004}?$$

(Hint: Compare with $(\sqrt{3}-\sqrt{2})^{2004}$.)

Problem 4.(Rubik Hypercube) Imagine an n-dimensional 3 by 3 by 3 ... by 3 hypercube, consisting of 3^n cells, smaller cubes. How many diagonals does it have? Here, a *diagonal* is defined to be a straight line consisting of 3 distinct cells. Note that you already solved the $n = 3$ case of this problem in the main test, and the answer there was $f(3) = 49$.

Your answer should be a concise formula for the function $f(n)$ expressing the number of diagonals as a function of dimension n. The answer will be graded on elegance, in addition to correctness, of course.

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