



UGA High School Varsity Math Tournament
October 26, 2024

TEAM ROUND

TIME: 1 HOUR

LENGTH: 3 PROBLEMS

MAX SCORE 210 POINTS

70 POINTS FOR A CORRECT ANSWER.

No calculators, slide rules, or any other such instruments are allowed.

You do not have to provide proofs; only the answers matter.

Each problem is worth 70 points, for a total of 210 points.

Problem 2. Consider the sets $\{x \bmod 37, 10x \bmod 37, 100x \bmod 37\}$ for integers $x = 1, \dots, 36$. What is the sum of the medians of these distinct sets?

Here, for an integer x , we write $x \bmod 37$ to be the unique integer a with $0 \leq a < 37$ such that $x - a$ is divisible by 37.

Problem 3. A baseball diamond consists of four bases, named home, first base, second base, and third base. Normally you run from home, to first, to second, to third, and back to home. Suppose we have two baseball diamonds, called A and B , which share a home plate. Suppose Alice starts on home plate, and her friend Bob keeps track of how many laps she makes around each baseball diamond. However, any laps she runs in the wrong direction Bob counts as negative laps.

Alice has a peculiar method for running the bases. From any base, she will randomly select an adjacent base to run to. The fourth time Alice returns to home plate, what is the probability that Bob's count is 0 laps around diamond A and 0 laps around diamond B ?