By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): ____________________ Name (print): ____________________

Student Number: ____________________

Instructor's Name: ____________________ Class Time: ________________

Problem Number	Total Points Possible	Points Made
1	0	
2	25	
3	20	
4	10	
5	8	
6	10	
7	15	
8	10	
9	17	
10	13	
11	10	
12	10	
13	15	
14	15	
Total:	178	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.
- A calculator is not necessary, and answers should be given in a form that can be directly entered into a calculator. If you give a numerical value it should be to within one decimal place unless otherwise stated.
- Common identities:

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta),\\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: How well will SD Worx perform next year after losing Demi Volllering, Marlen Reusser, Niamh Fisher-Black, Christine Majerus, and Anna Shackley?

2. Determine all values of x that satisfy each equation below.

   (a) [5 pts] $\sqrt{4x+1} = \dfrac{1}{2}$.

   (b) [5 pts] $\dfrac{3}{2-x} = 5x$.

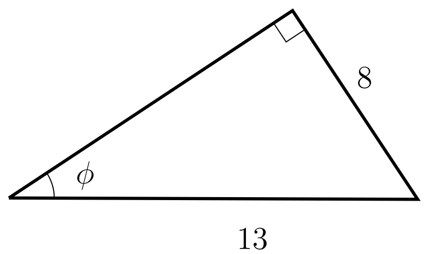
(c) [5 pts] $\log_3(3 + \sqrt{x}) = 4$.

(d) [5 pts] $(8 - e^x)^5 = 32$.

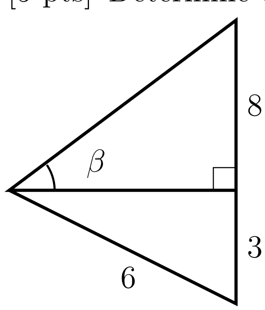
(e) [5 pts] $4 \cdot 13^{2-x} = 14 \cdot 7^{x-1}$.

3. Determine the value of each of the requested quantities below. If an exact number is not requested, numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine its numerical value**.)

   (a) [5 pts] Determine the sine, cosine, and tangent of the angle ϕ as shown in the diagram below:



   (b) [5 pts] Determine the radian measure of the angle β in the diagram below.



(c) [5 pts] Verify the equality

$$(\cos(\theta) + \sin(\theta))^2 + (\cos(-\theta) + \sin(-\theta))^2 \quad = \quad 2.$$

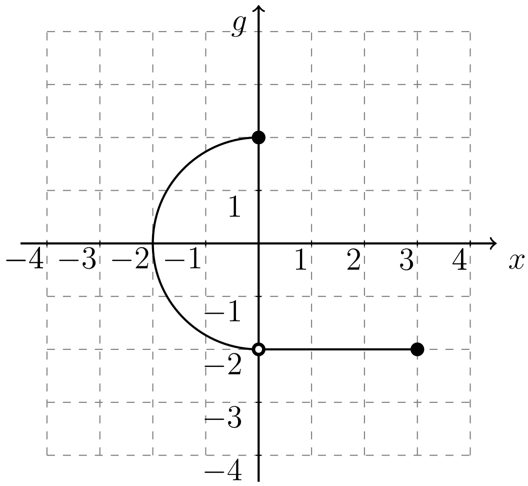
(Show every step, and do not make more than one algebraic step at a time.)

(d) [5 pts] Determine the **exact** numerical value of the following expression and express it without the use of any trigonometric functions:

$$\sin\left(\arctan(0.1) + \arccos(-0.4)\right).$$

(Show your work and do not provide a numerical estimate or use a calculator.)

4. Three relationships, g, h, and k, are defined below. Use these definitions to answer each of the questions that follow.



x	h
-1	2
0	5
1	3
2	3

$$k(x) \quad = \quad x^2 + 1.$$

(a) [5 pts] For each relationship (g, h, and k) determine whether or not the relationship is a function. Briefly justify your conclusion. (If you use a "named" test, state the name of the test and explicitly discuss how it implies the relationship is a function.)

g

h

k

(b) [5 pts] For each relationship that is a function, determine if the function is one-to-one. (Briefly justify your conclusion and do not simply state the name of a test.)

g

h

k

5. Two functions are defined by

$$\begin{aligned} m(x) &= 4^x + 1, \\ p(x) &= (1+b)^x. \end{aligned}$$

(a) [5 pts] Determine the domain and range of $m(x)$. State any asymptotes for the function $m(x)$.

(b) [3 pts] Determine the values of b that cause $p(x)$ to be an exponentially decaying function. (Your answer should be an interval. Use interval notation.)

6. Answer each question below using the function

$$r(x) \quad = \quad \frac{6}{2x+1}.$$

(a) [5 pts] Determine the inverse of the function.

(b) [5 pts] Determine the average rate of change of the function, $r(x)$, from $x = 0$ to $x = 5$.

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] The total mass of phytoplankton in an area of the ocean increases at a constant rate each day. *The total mass of the phytoplankton as a function of time over a month.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

   (b) [5 pts] The total mass of phytoplankton in an area of the ocean increases by 1.3% each week. *The total mass of the phytoplankton as a function of time over several months.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

   (c) [5 pts] The total mass of phytoplankton in an area of the ocean increases during the summer months and decreases during the winter months and repeats this cycle each year. *The total mass of the phytoplankton as a function of time over many years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

8. [10 pts] Doctor Bunsen Honeydew has a test tube with 0.7mg of a sensitive material in it, and he begins to gently sprinkle more of the chemical into the test tube at a constant rate of 0.15mg per minute. The test tube can hold 3.0mg of the material. Determine a function that returns the amount of material in the test tube given the time in minutes, and determine when the test tube starts to overflow. (Your function for the amount of material should only be for times before an overflow occurs.)

9. Each of the following questions refer to the function

$$d(x) \quad = \quad \begin{cases} -(x-1)^2+1 & 0 \;\le\; x \;\le\; 1.5, \\ -(x-2)^2+1 & 1.5 \;<\; x \;\le\; 3.0, \end{cases}$$

(a) [5 pts] Determine the coordinates for all local maximum values of $d(x)$. (Briefly explain why your coordinates are a local maximum.)

(b) [5 pts] Determine the intervals where the function, $d(x)$, is increasing. (Your answer should be in interval notation.)

(c) [5 pts] Determine the domain and range of the function, $d(x)$.

(d) [2 pts] Is it possible to draw a sketch of the graph without lifting the pencil from the page?

10. A function is defined to be

$$s(x) \quad = \quad -3e^{rx} + C$$

where $s(0) = 5$, and $s(6) = 7.5$.

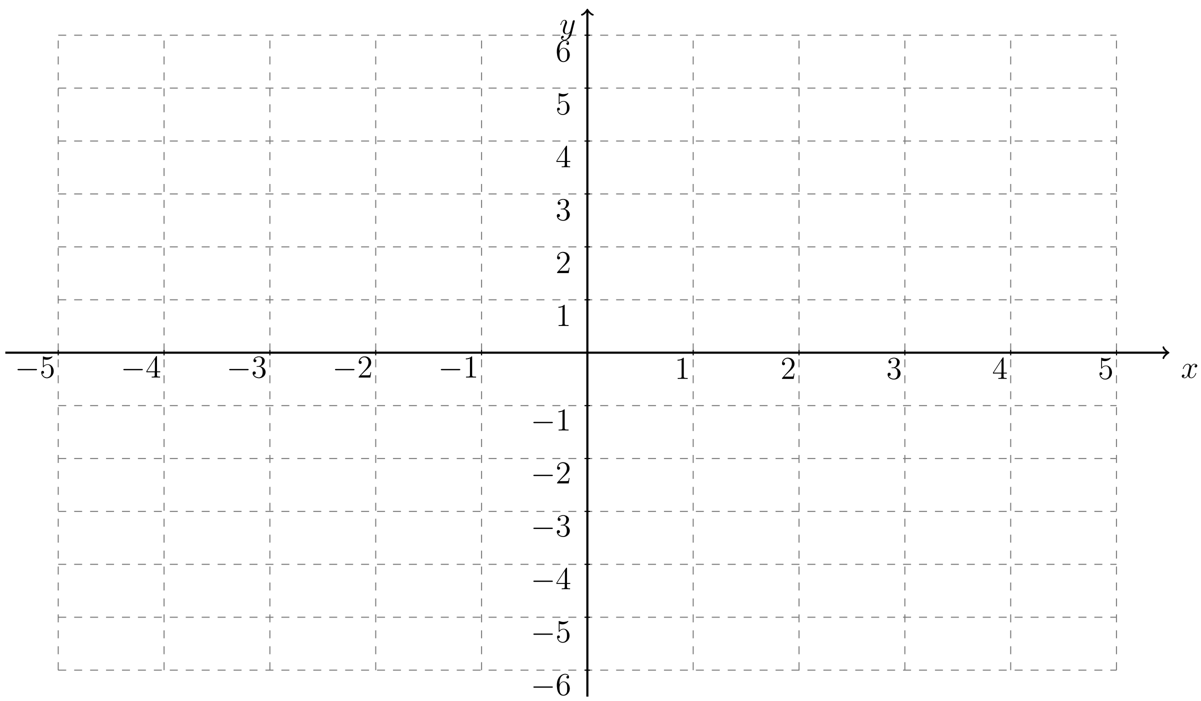
(a) [5 pts] Determine the values of the constants C and r.

(b) [5 pts] Is the function always increasing, always decreasing, or neither? (Briefly justify your answer.)

(c) [3 pts] Determine if the value of the function approaches a particular number as the value of x gets extremely large, and if so, determine the value.

11. [10 pts] Use the axes below to make a sketch of the graph of the function

$$w(x) \quad = \quad 3\sin\left(\frac{\pi}{3}x + \frac{4\pi}{3}\right) - 2.$$



12. A kudzu vine grows 3cm a day, and it climbs straight up the side of a building. A video camera will be placed on the ground directly in front of the vine. The camera will be 150cm away from the wall. The vine starts at 10cm above the ground when the camera is put in place. The angle of elevation for the camera will be constantly adjusted so that the top of the vine is in the center of the picture.

    (a) [5 pts] When the angle of elevation reaches $\frac{4\pi}{9}$ the height of the vine will be measured, and whoever has the closest guess will win a prize. What will the height be when this occurs?

    (b) [5 pts] Determine the angle of elevation as a function of time in days.

13. It is estimated[1] that the distance a locust can hop is related to the length of the animal's femur,

$$d = C \cdot l^{2/3},$$

where d is the distance of the hop in meters(m), C is a constant, and l is the length of the animal's femur in millimeters(mm). The length of a locust's femur (mm) is related to the animal's total mass,

$$l = 6 \cdot m^{1/3},$$

where m is the animal's mass in grams.

(a) [5 pts] The length of a given locust's femur is 7.5mm, and the distance it can hop is measured to be 0.05m. Determine the value of C.

(b) [5 pts] The distance that a different locust can hop is 0.06m. Use the relationships above and your estimate of C to determine the length of its femur. (If you do not feel confident in your calculation of C above, use a value of $C = 0.025$.)

(c) [5 pts] Determine the function that returns the mass of a locust given the distance it can hop. (Use the same value for C that you used above.)

[1] Julie M. Gabriel; The Development of the Locust Jumping Mechanism : I. Allometric Growth and its Effect on Jumping Performance. J Exp Biol 1 September 1985; 118 (1): 313-326.

14. [15 pts] An insect will move 100mm. The insect can either walk or it can run. The energy expenditure depends on whether it walks or runs.

Walking The energy expenditure when walking is the square of the distance walking. (i.e. $(\text{distance walking})^2$). For example, if the insect walks 3mm, the expenditure is 9 units.

Running The energy expenditure when running is six times the square of the distance running. (i.e. $6(\text{distance running})^2$). For example, if the insect runs 3mm, the expenditure is 54 units.

Determine the distance the insect should walk and the distance it should run to **minimize** the insect's total energy cost.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): ________________ Instructor (print): ________________ Time: ________________