By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Points Possible	Points Made
1	0	
2	22	
3	20	
4	10	
5	13	
6	20	
7	15	
Total:	100	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
- Common identities:

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta),\\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: What will Grace Brown do in her retirement given her wins at the Olympics and the World Championships?

2. Determine all of the values of x for each question below that satisfy the given equation. If no values of x satisfy the equation provide a brief justification as to how you arrived at your conclusion.

_____ (a) [5 pts] $e^{3x+1} = 7$.

(b) [5 pts] $\dfrac{e^{2x}}{5 - e^{2x}} = 42$.

_____

(c) [5 pts] $\ln(1 - 8x) = 7$.

(d) [7 pts] $5 \cdot 9^{2x} = 4 \cdot 13^{x+1}$.

3. Simplify each of the following functions to an equivalent expression that contains no exponentials and no logarithms.

(a) [5 pts] $\log\left(\dfrac{1}{100^w}\right)$

(b) [5 pts] $e^{3\ln(u)-\ln(5v)}$

(c) [5 pts] $\log_5\left(\dfrac{5^u \cdot 25^w}{(5^v)^2}\right)$

(d) [5 pts] $\left(\dfrac{1+9^{2\log_9(u+4)}}{9^{1+\log_9(w)}}\right)$

4. The questions below refer to the function

$$p(x) = \frac{1}{x+1} + 8.$$

(a) [5 pts] Show that $p(x)$ is a one-to-one function. (Show every step.)

(b) [5 pts] Determine the inverse of $p(x)$.

5. The number of lobsters in a fishery are estimated, and the estimated population is recorded. An index is reported to local managers, and the index for any time, in days after January 1, is calculated using

$$\text{Index}(t) \;=\; \log_{10}\left(\frac{\text{population at time } t}{\text{population on January 1}}\right).$$

The number of lobsters on January 1 is estimated to be 20,000 animals.

(a) [3 pts] What is the current index if the current number of animals is estimated to be 40,000 animals?

(b) [5 pts] If the index is -0.2 what is the estimated number of lobsters?

(c) [5 pts] If the population of lobsters is estimated to be $P(t) = 20,000e^{0.01t}$, how long will it take until the index is equal to 1.3?

6. A marble is dropped into a container of very thick molasses. The depth in centimetres, below the surface of the molasses, is given by

$$d(t) = 3 - 3e^{-0.2t},$$

where t is the time in seconds after the marble is released.

(a) [5 pts] What will the depth of the marble be after four seconds?

(b) [5 pts] What depth will the marble get close to after a very long time? (Briefly justify your result.)

(c) [5 pts] Determine the function that provides the time after the marble has dropped given the depth of the marble.

(d) [5 pts] Determine the time it will take for the marble to sink to a depth of 95% of the value you obtained in part b.

7. A sample initially has 3,000 bacteria, and the number of bacteria are approximated using an exponential function,

$$P(t) = Ce^{rt},$$

where t is the time in hours.

(a) [5 pts] After 24 hours it is estimated that there are 4,000 bacteria in the sample. Determine the values of C and r.

(b) [5 pts] How long does it take for the population to double?

(c) [5 pts] **After** 48 hours from the start a disinfectant is introduced into the sample causing the number of bacteria to decrease like $P(t) = Ae^{w \cdot t}$, where A and w are constants. It is estimated that after 72 hours there are 2,000 bacteria in the sample. Determine the values of w and A.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____