University of Georgia   Math 1113
Department of Mathematics   **Test 1**   Fall 2024

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): _____   Name (print): _____

Student Number: _____

Instructor's Name: _____   Class Time: _____

- If you need extra space use the last page.

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.

- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.

- Please turn off your mobile phone.

- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

Problem Number	Points Possible	Points Made
1	0	
2	20	
3	20	
4	15	
5	20	
6	15	
7	10	
8	10	
Total:	110	

1. [2 Bonus] Common Knowledge: What is the smallest ever winning margin of victory in the Tour de France?

2. Determine all of the values of x for each question below that satisfy the given equation. If no values of x satisfy the equation provide a brief justification as to how you arrived at your conclusion.

_____ (a) [5 pts] $\dfrac{3x}{4x-1} = 2$.

(b) [5 pts] $\sqrt{x+1} = x - 3$.

_____

(c) [5 pts] $\dfrac{3}{\sqrt{x+1}} = 2$.

_____

(d) [5 pts] $|2x - 1| = 3$.

_____

A

Points earned: _____
out of a possible 20 points

3. Three functions, $g(x)$, $h(x)$ and $k(x)$, are given below. Use the functions to answer each of the following questions. Any intervals should be stated using interval notation and not inequalities. If a quantity does not exist provide a brief justification as to how you arrived at your conclusion.

$$g(x) = \frac{x+5}{\sqrt{x^2-16}}, \qquad\qquad h(x) = \begin{cases} -x & x < 0, \\ x^2 & x \geq 0. \end{cases}$$
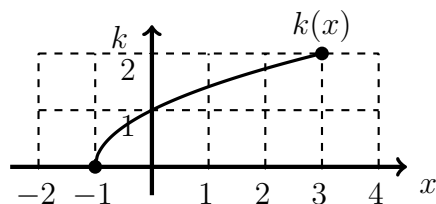
(a) [5 pts] Determine the domain of $g(x)$.

(b) [5 pts] Determine the range of $h(x)$.

(c) [5 pts] Determine the domain and range of $k(x)$.

(d) [5 pts] Determine the value of $k(h(-4))$.

Points earned: _____
out of a possible 20 points

4. Determine equations for each of the lines in the descriptions that follow.

   (a) [5 pts] The line that includes the points $(5, 1)$ and $(-3, 2)$.

   (b) [5 pts] The line that includes the point $(8, -1)$ and is parallel to the line $4x + 7y = 9$.

   (c) [5 pts] The vertical line that includes the point $(4, 2)$.

5. Answer each question below, and the function $q(x)$ is defined to be

$$q(x) = x^2 + 2x + 2.$$

(a) [5 pts] Use the axes below to make a rough sketch of the function. State any intervals where the function is increasing, and state any intervals where the function is decreasing.
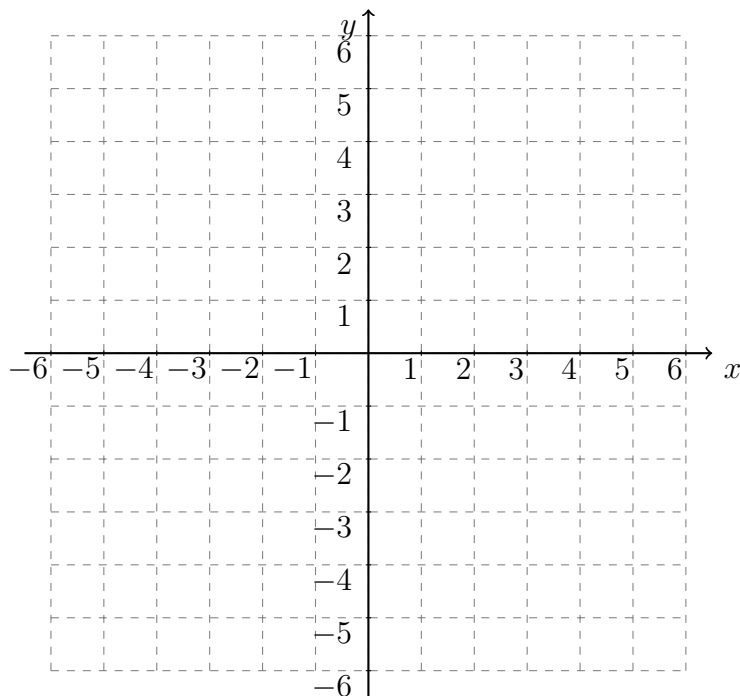
(b) [5 pts] Determine the x and y intercepts of the function. If the function does not have an intercept briefly justify your conclusion.

(c) [5 pts] Determine the average rate of change of $q(x)$ from $x = -1$ to $x = 3$.

(d) [5 pts] Determine all possible values of a so that the average rate of change of the function from $x = 0$ to $x = a$ is one.

6. A tortoise lives in a designated area and digs tunnels. The tortoise digs 0.6 meters of new tunnel every 24 hours. It is estimated that 0.2 meters of the tunnel collapse every 24 hours. At the start of a study there are 2.6 meters of tunnel on the site.

   (a) [5 pts] Determine the function that gives the total length of new tunnels dug by the tortoise given the number of **hours** from the start of the study. (Ignore the length of tunnels that have collapsed and ignore the pre-existing tunnels.)

   (b) [5 pts] Determine a function that gives the total length of tunnels on the site given the number of hours from the start of the study. (Include the length that collapsed and include the pre-existing tunnels.)

   (c) [5 pts] Shoco owls use the tortoise's tunnels for nesting. It is estimated that the number of Shoco owls on the site is related to the length of tunnels available, and is given by

   $$S(x) \;=\; \begin{cases} 0 & x \;<\; 4, \\ \sqrt{x} - 2 & x \;\geq\; 4, \end{cases}$$

   where x is the length of tunnels in meters. Determine the function that gives the number of Shoco owls on the site as a function of the number of hours since the start of the study.

7. [10 pts] The graph of a function, $w(x)$, is shown in the diagram below, and the second set of axes has a graph of the function $u(x) = a \cdot w\,(b \cdot x + c) + d$. Determine the values of a, b, c, and d.
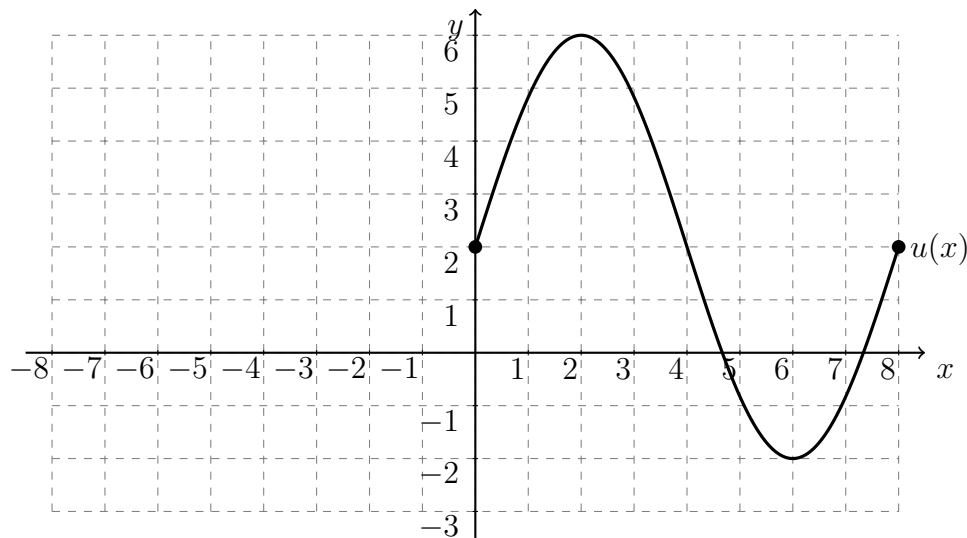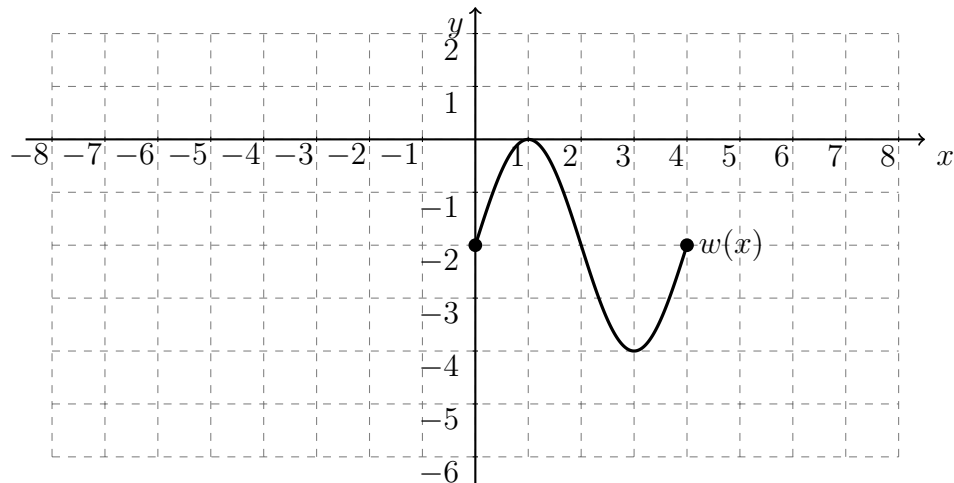
$a:$

$b:$

$c:$

$d:$

8. [10 pts] A biological system has two trophic levels. The first level will have x kg of mass, and the second level will have y kg of mass. The whole system can support a total of 100 kg of mass. The efficiency of each level depends on the mass within the system:

Efficiency of Level 1 The efficiency of the first level is $120 - x$.

Efficiency of Level 2 The efficiency of the second level is $150 - y$.

The total efficiency of the system is the product of the two individual efficiencies. Determine the mass for each level that will maximize the total efficiency of the whole system.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____